NAG Toolbox for MATLAB

f07ts

1 Purpose

f07ts solves a complex triangular system of linear equations with multiple right-hand sides, $AX = B$, $A^{\mathrm{T}}X = B$ or $A^{\mathrm{H}}X = B$.

2 Syntax

```
[b, info] = f07ts(uplo, trans, diag, a, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07ts solves a complex triangular system of linear equations $AX = B$, $A^{\mathrm{T}}X = B$ or $A^{\mathrm{H}}X = B$.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Higham N J 1989 The accuracy of solutions to triangular systems *SIAM J. Numer. Anal.* **26** 1252–1265

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo – string**

Indicates whether A is upper or lower triangular.

uplo = 'U'

A is upper triangular.

uplo = 'L'

A is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **trans – string**

Indicates the form of the equations

trans = 'N'

The equations are of the form $AX = B$.

trans = 'T'

The equations are of the form $A^{\mathrm{T}}X = B$.

trans = 'C'

The equations are of the form $A^{\mathrm{H}}X = B$.

Constraint: **trans** = 'N', 'T' or 'C'.

3: **diag – string**

Indicates whether A is a nonunit or unit triangular matrix.

diag = 'N'

A is a nonunit triangular matrix.

diag = 'U'

A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: **diag** = 'N' or 'U'.

4: **a**(**lda**,∗) – **complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n triangular matrix A.

If **uplo** = 'U', A is upper triangular and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', A is lower triangular and the elements of the array above the diagonal are not referenced.

If **diag** = 'U', the diagonal elements of A are assumed to be 1, and are not referenced.

5: **b**(**ldb**,∗) – **complex array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n** – **int32 scalar**

Default: The second dimension of the array **a**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p** – **int32 scalar**

Default: The second dimension of the array **b**.

r, the number of right-hand sides.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb

5.4 Output Parameters

1: **b**(**ldb**,∗) – **complex array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r solution matrix X.

2: **info** – **int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **trans**, 3: **diag**, 4: **n**, 5: **nrhs_p**, 6: **a**, 7: **lda**, 8: **b**, 9: **ldb**, 10: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, $a(i,i)$ is exactly zero; A is singular and the solution has not been computed.

7 Accuracy

The solutions of triangular systems of equations are usually computed to high accuracy. See Higham 1989.

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A+E)x = b$, where

$$|E| \leq c(n)\epsilon|A|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A,x)\epsilon, \qquad \text{provided} \qquad c(n)\,\mathrm{cond}(A,x)\epsilon < 1,$$

where $\mathrm{cond}(A,x) = \left\| |A^{-1}||A||x| \right\|_\infty / \|x\|_\infty$.

Note that $\mathrm{cond}(A,x) \leq \mathrm{cond}(A) = \left\| |A^{-1}||A| \right\|_\infty \leq \kappa_\infty(A)$; $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$ and it is also possible for $\mathrm{cond}\left(A^{\mathrm{H}}\right)$, which is the same as $\mathrm{cond}\left(A^{\mathrm{T}}\right)$, to be much larger (or smaller) than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling f07tv, and an estimate for $\kappa_\infty(A)$ can be obtained by calling f07tu with **norm_p** $=$ 'I'.

8 Further Comments

The total number of real floating-point operations is approximately $4n^2r$.

The real analogue of this function is f07te.

9 Example

```
uplo = 'L';
trans = 'N';
diag = 'N';
a = [complex(4.78, +4.56), complex(0, 0), complex(0, 0), complex(0, 0);
      complex(2, -0.3), complex(-4.11, +1.25), complex(0, 0), complex(0,
0);
       complex(2.89, -1.34), complex(2.36, -4.25), complex(4.15, +0.8),
complex(0, 0);
     complex(-1.89, +1.15), complex(0.04, -3.69), complex(-0.02, +0.46),
complex(0.33, -0.26)];
b = [complex(-14.78, -32.36), complex(-18.02, +28.46);
```

```
     complex(2.98, -2.14), complex(14.22, +15.42);
     complex(-20.96, +17.06), complex(5.62, +35.89);
     complex(9.54, +9.91), complex(-16.46, -1.73)];
[bOut, info] = f07ts(uplo, trans, diag, a, b)
```

```
bOut =
  -5.0000 - 2.0000i   1.0000 + 5.0000i
  -3.0000 - 1.0000i  -2.0000 - 2.0000i
   2.0000 + 1.0000i   3.0000 + 4.0000i
   4.0000 + 3.0000i   4.0000 - 3.0000i
info =
           0
```